How Complicated is Minesweeper?

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Minesweeper

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- When skilfully played, the task can be completed without having to take many risky guesses!

- In fact the complexity of minesweeper is related to an important unsolved problem in mathematics, which in turn relates to cracking codes on the internet.
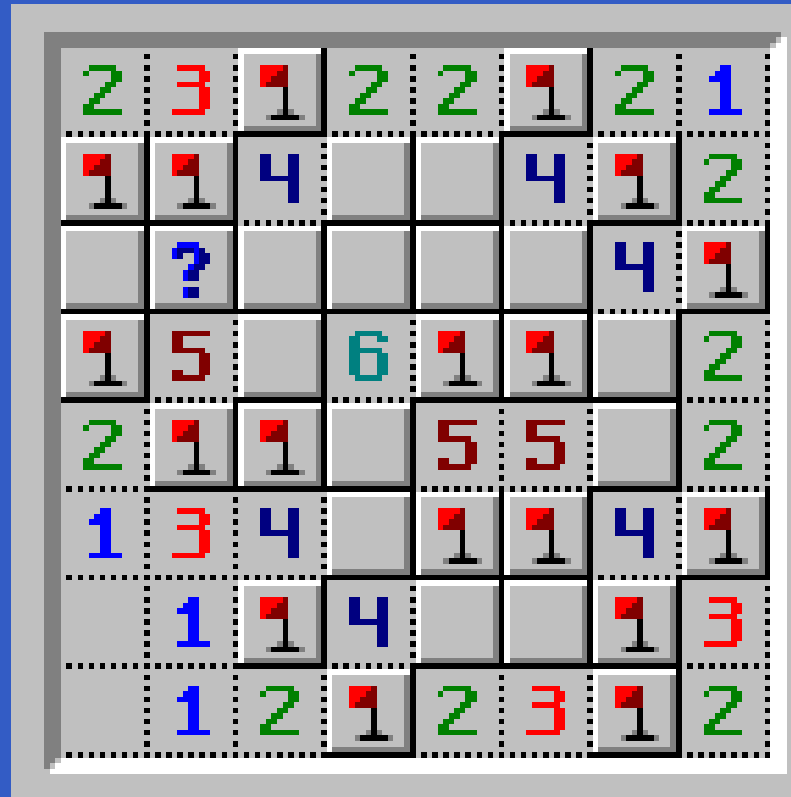
Playing the game

Does (2,6) have a mine?
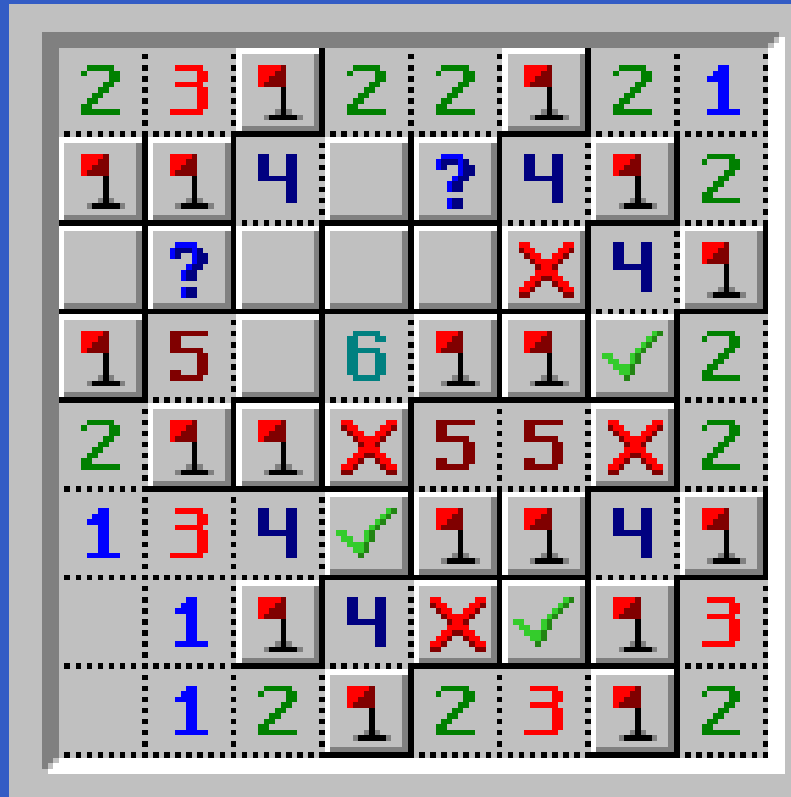
Playing the game

These must have mines

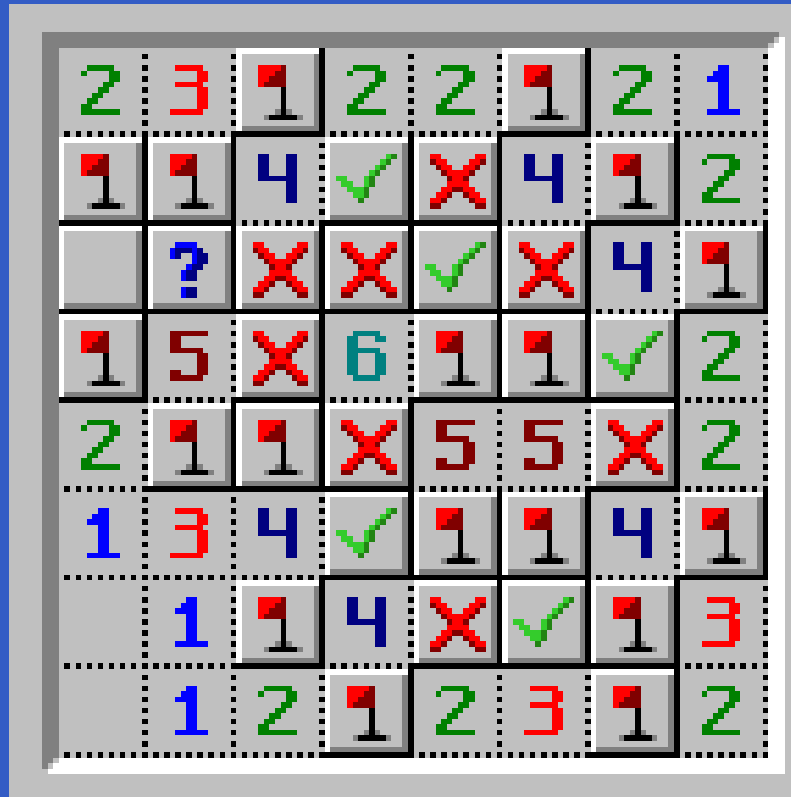# Playing the game

Does (5,2) have a mine?
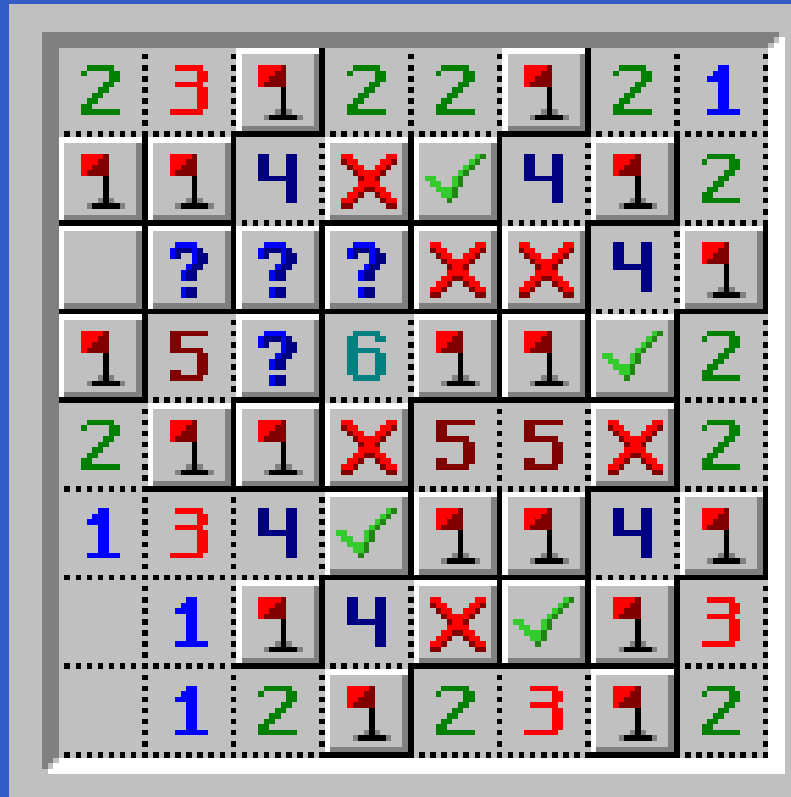
Playing the game

If so, does (5,7) have a mine?

Playing the game

If (5,2) and (5,7) have mines

Playing the game

If (5,2) has a mine but (5,7) hasn't

Playing the game

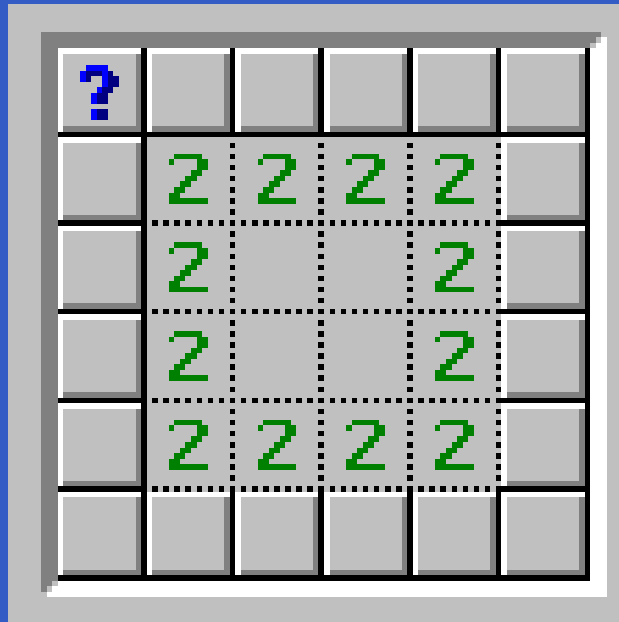If (5,2) is clear

Playing the game
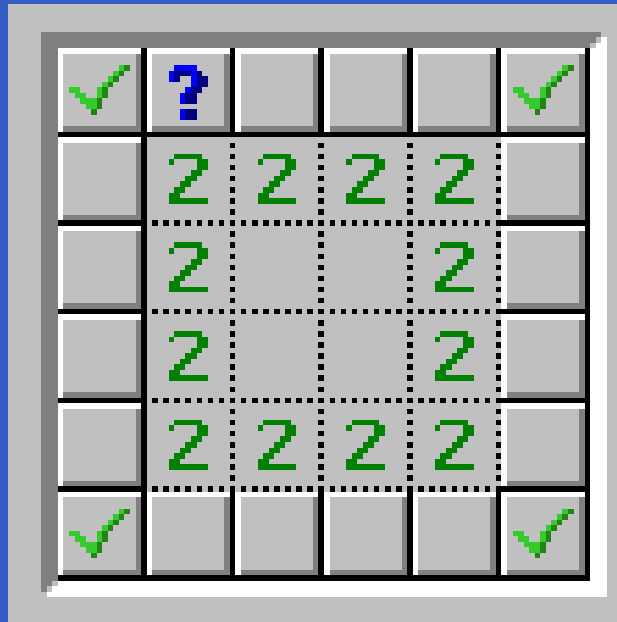

So (2,6) is clear!
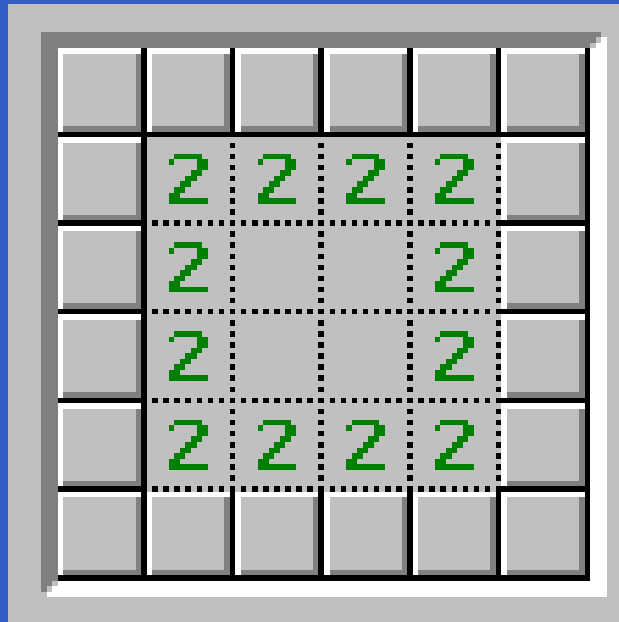
A puzzle

Determine the location of all mines!
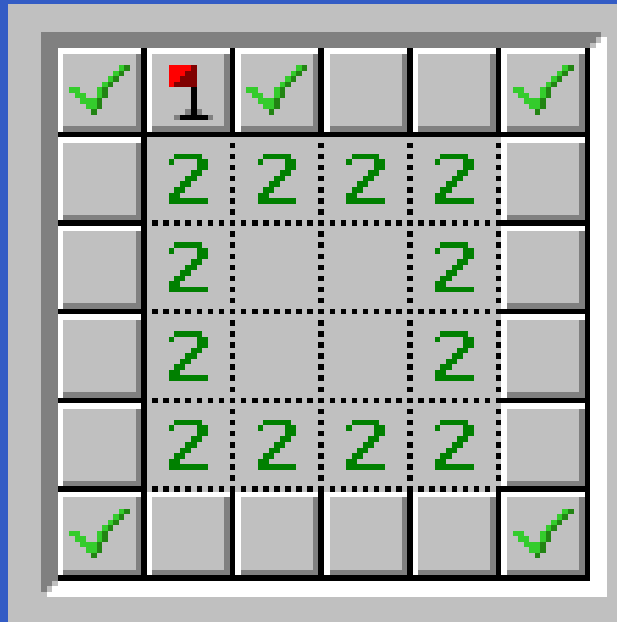
A puzzle

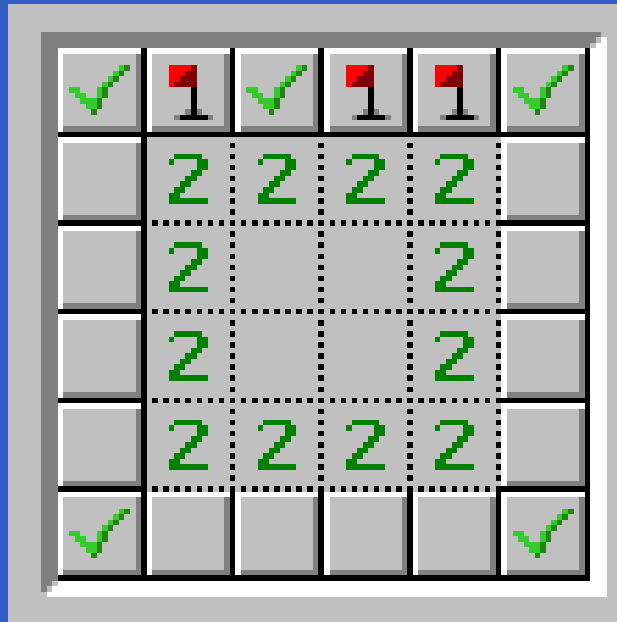Can this be a mine?

A puzzle

If this is a mine the one next to it is clear…

A puzzle

So. . .

A puzzle

. . . which is impossible!
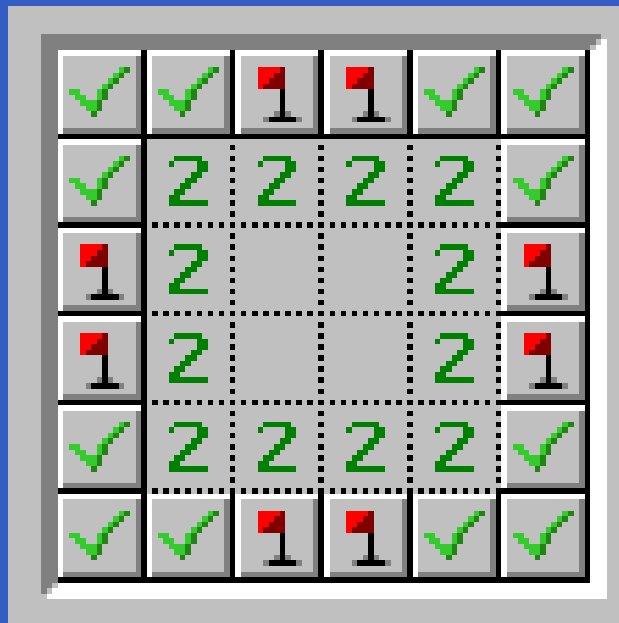
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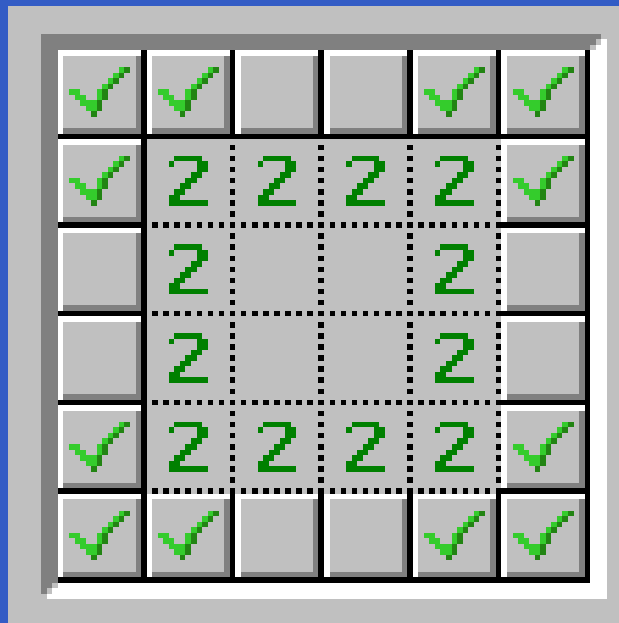
Therefore. . .

Solved!

What is the question?

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- *We want to play in such a way that we never make a risky move when there is some square which can be uncovered safely.*

The consistency problem

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- To determine if a square is free of any mines change the configuration marking it with a mine. Then ask if the result is consistent. If not, it is safe to clear the square!

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- There *is* an algorithm that solves this problem and solves the consistency problem. All you need to do is go though all the different combinations for the mines on the minefield in turn.

So what's the difficulty?

- Typical minesweeper configurations have many squares. (The so-called 'advanced' board has 99 mines in 480 squares, that's

  560220999337421345429058985 7758...95706311681983856732951596334 81600

  or over 10^{100} possible ways of placing these 99 mines.)

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- No known method can search through all these possibilities in reasonable time.

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- However, it seems unlikely that such an algorithm exists.

A wire
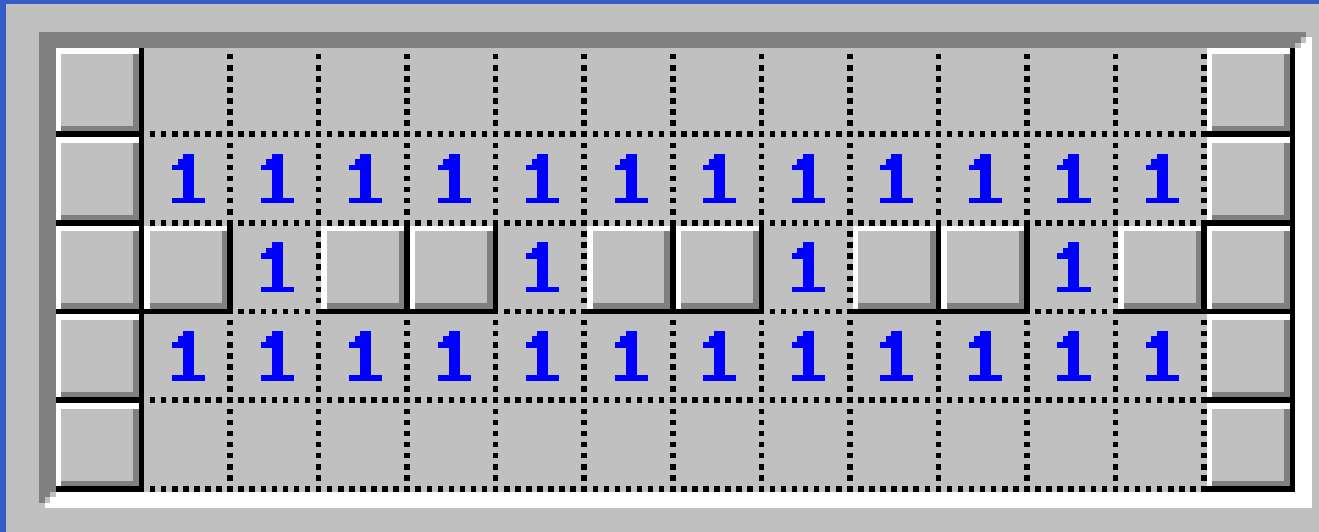
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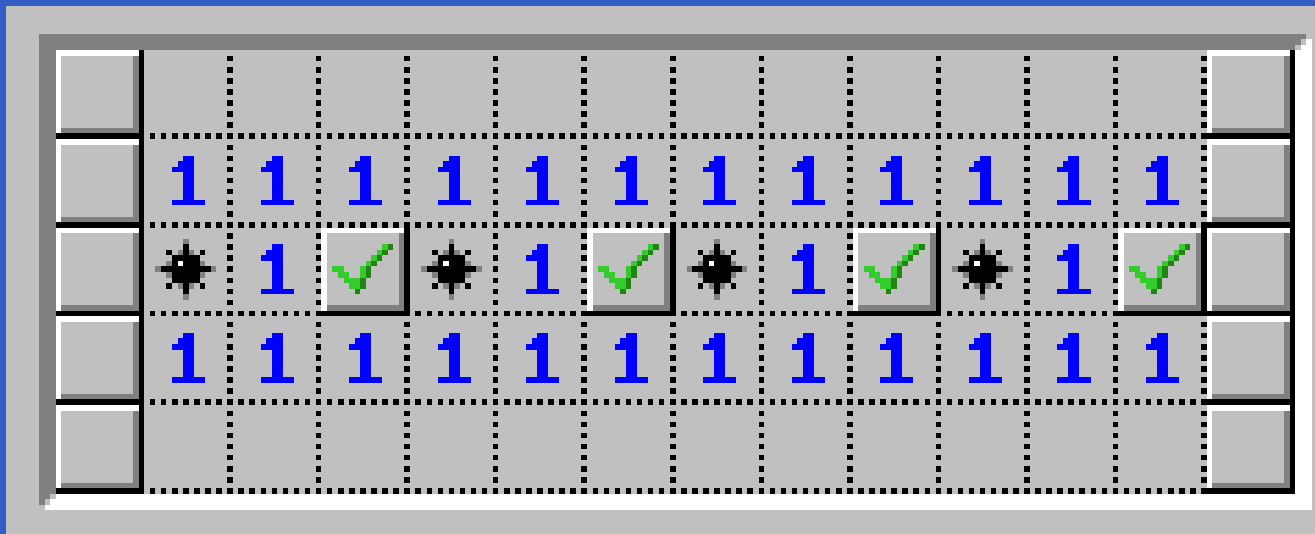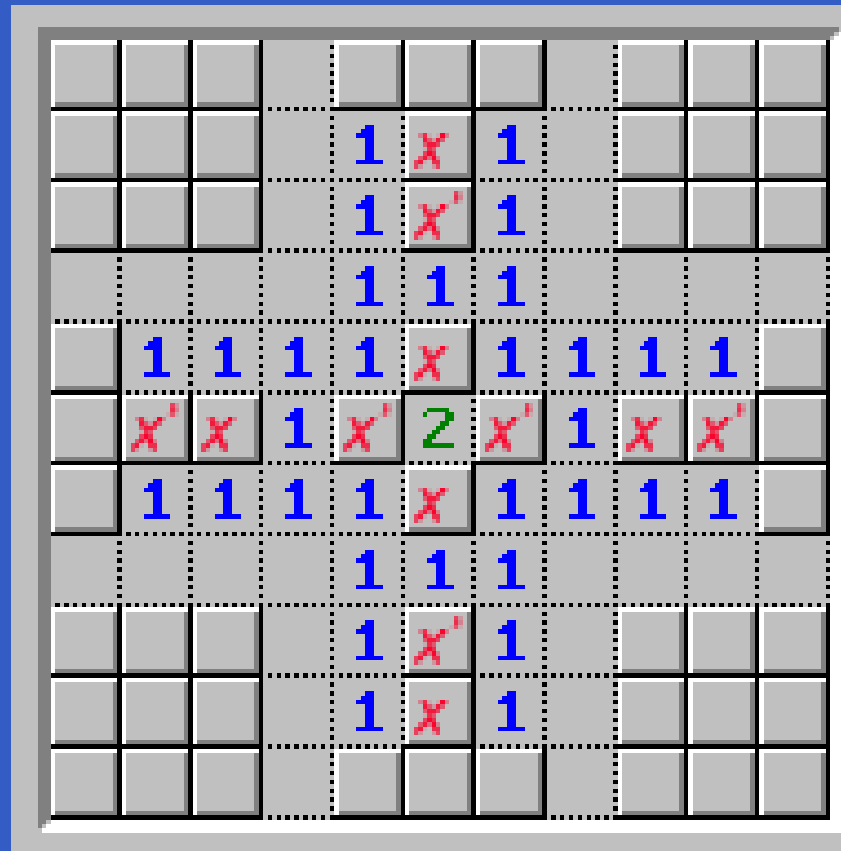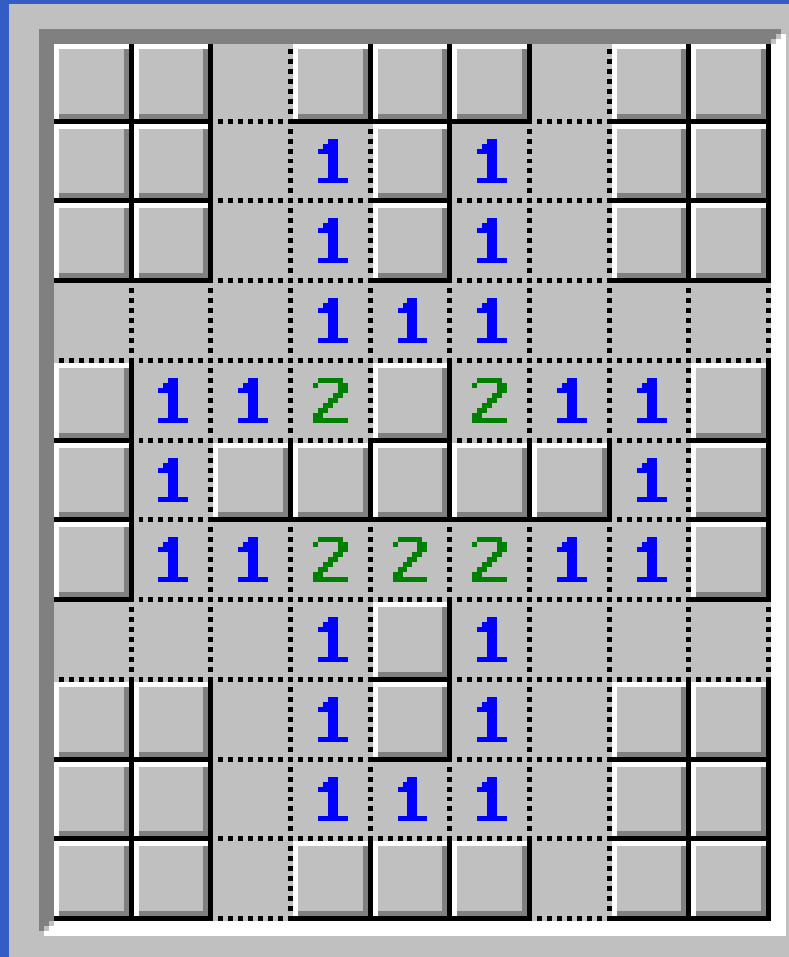
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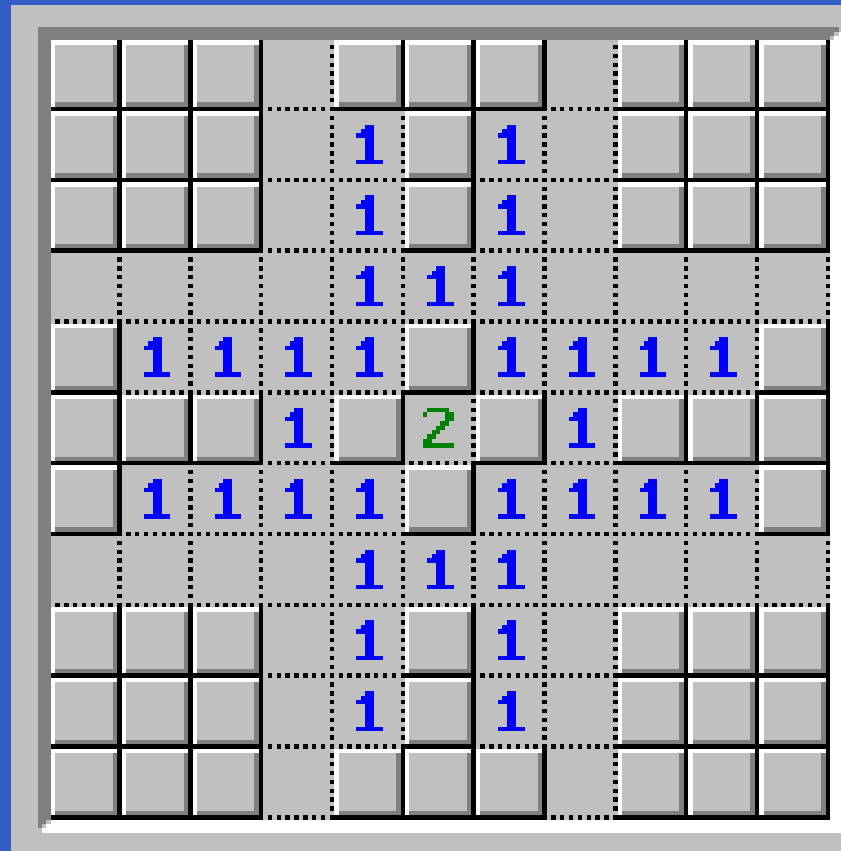
Use this to split, bend, or 'invert' wires.
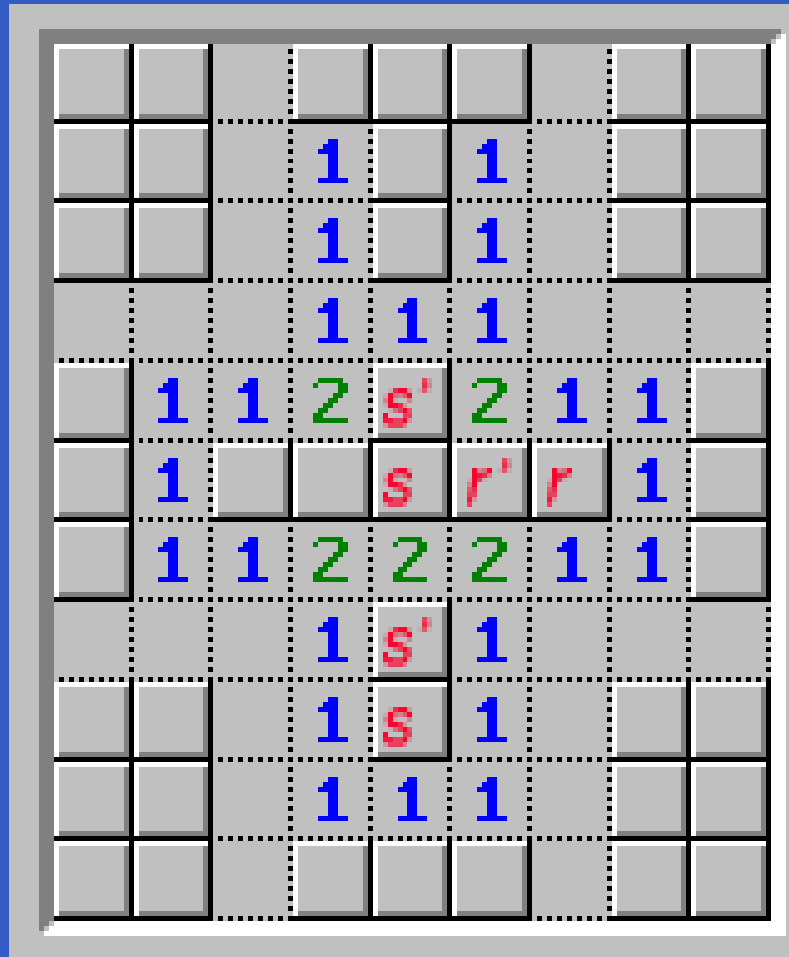
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A crossover

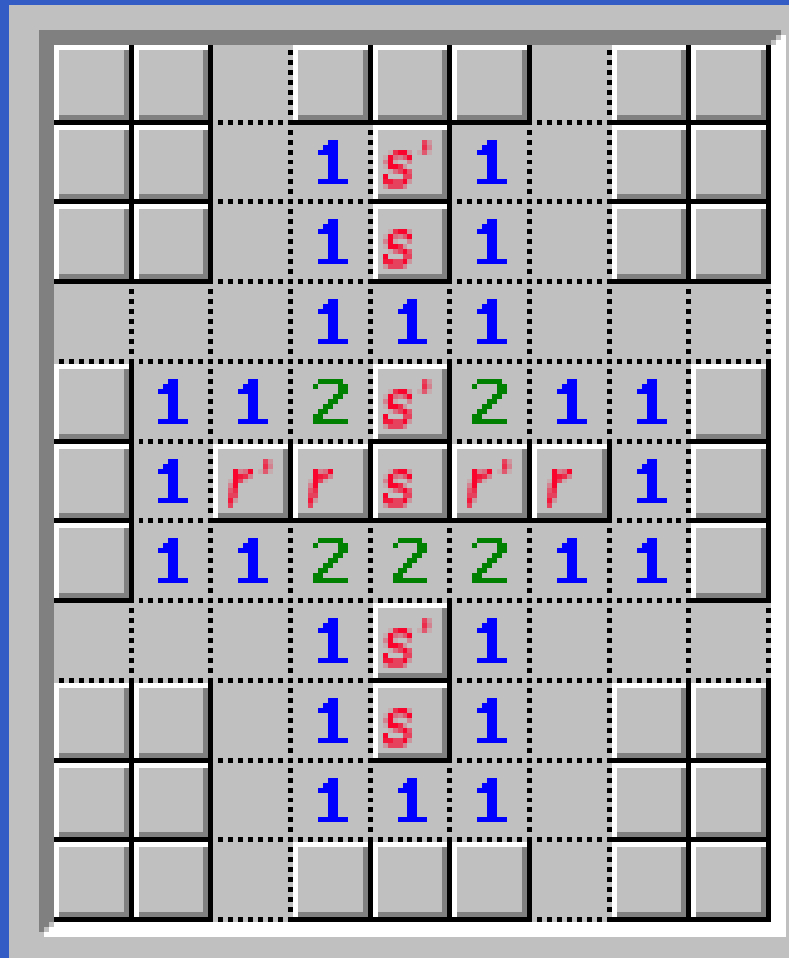Use this to cross two wires over!
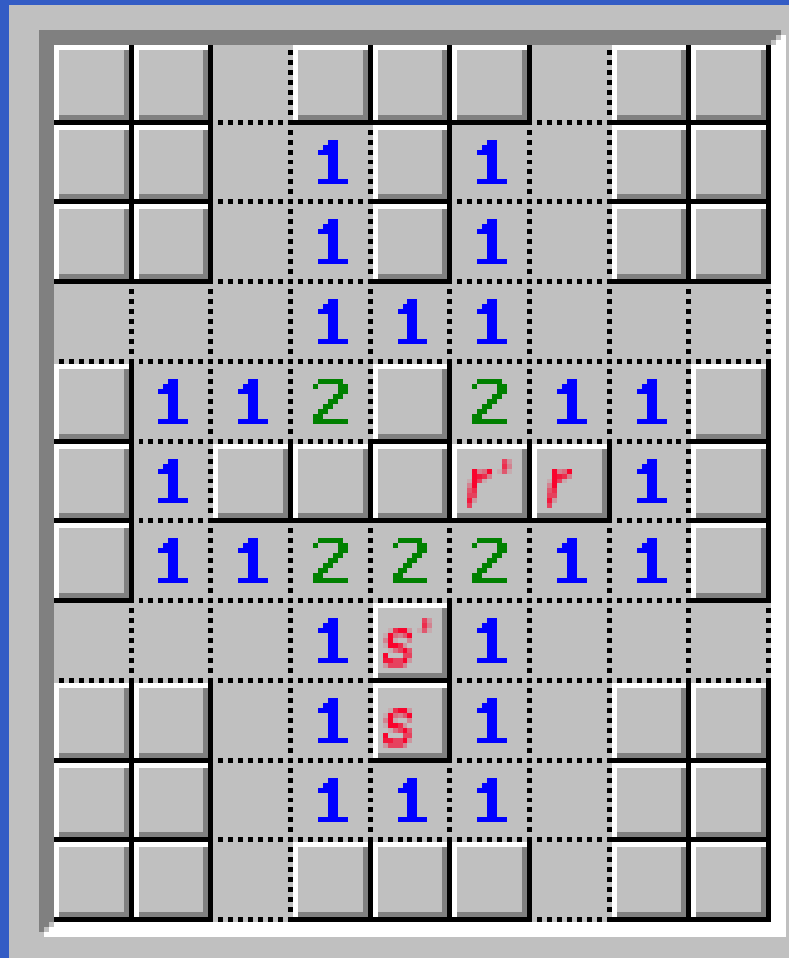
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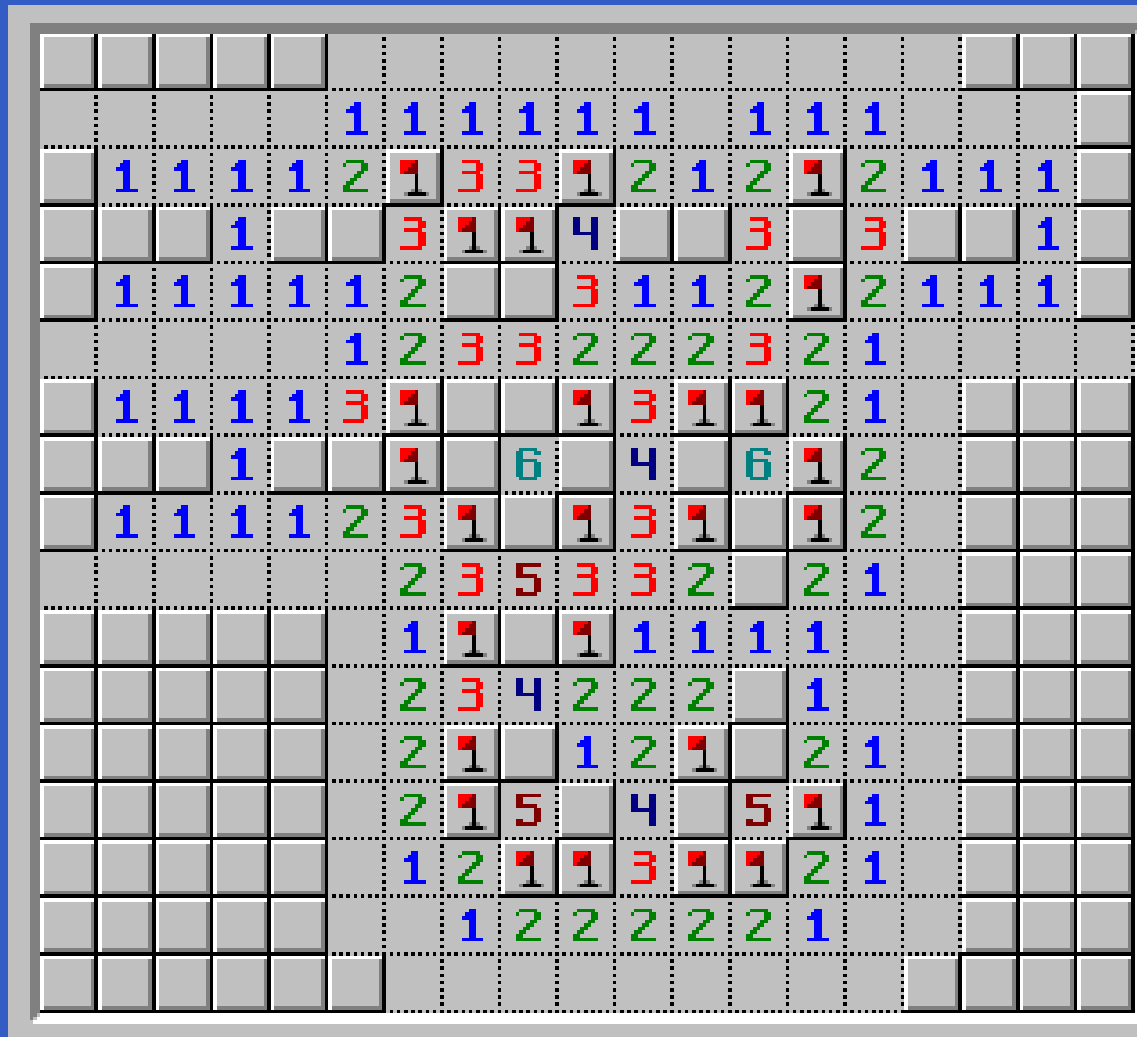Use this to cross two wires over!

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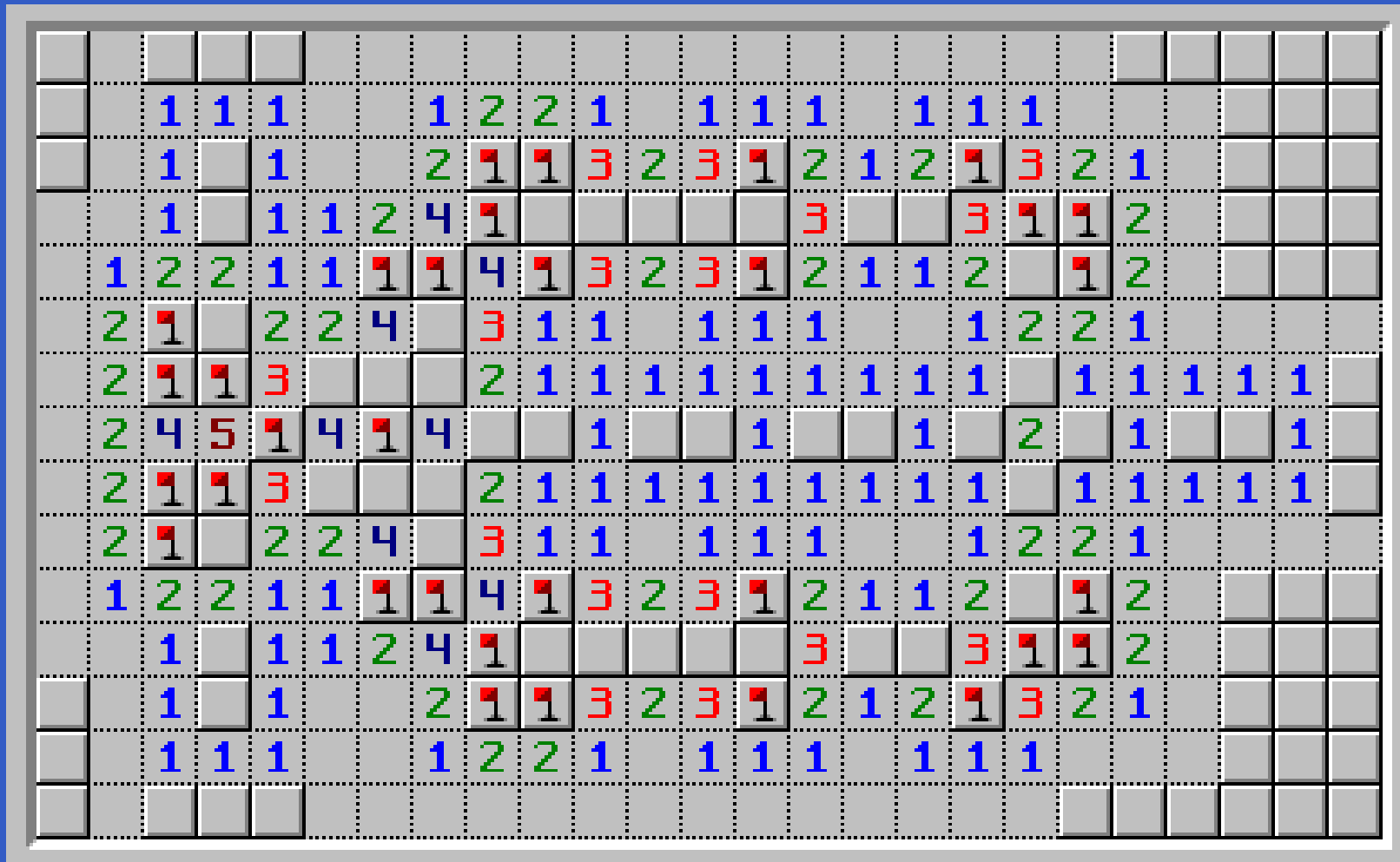

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An XOR gate

An XOR gate

An XOR gate

An AND gate

An AND gate. . .

An AND gate

... and its internal wires.

An AND gate

If *t* is a mine…

An AND gate

If *t* is not a mine. . .

An OR gate
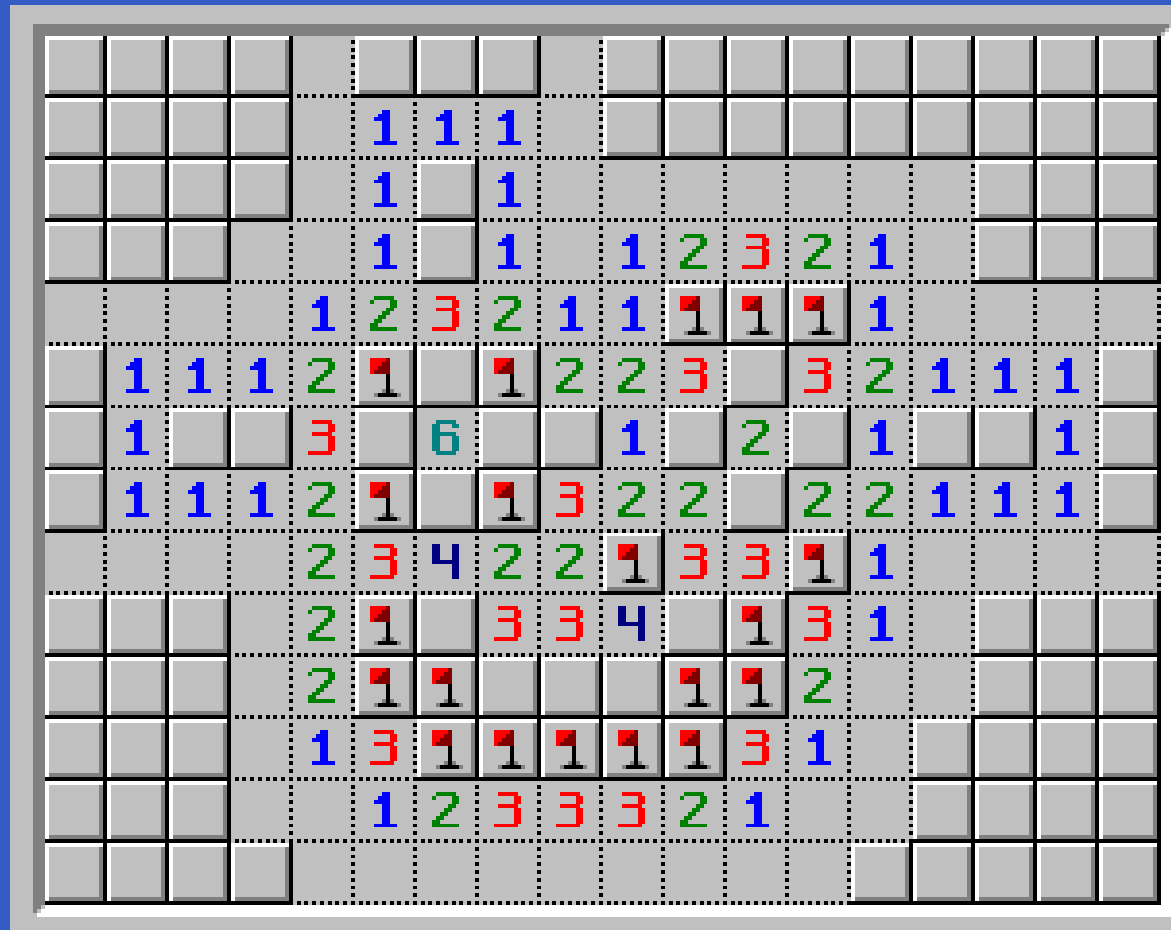
An OR gate...

An OR gate
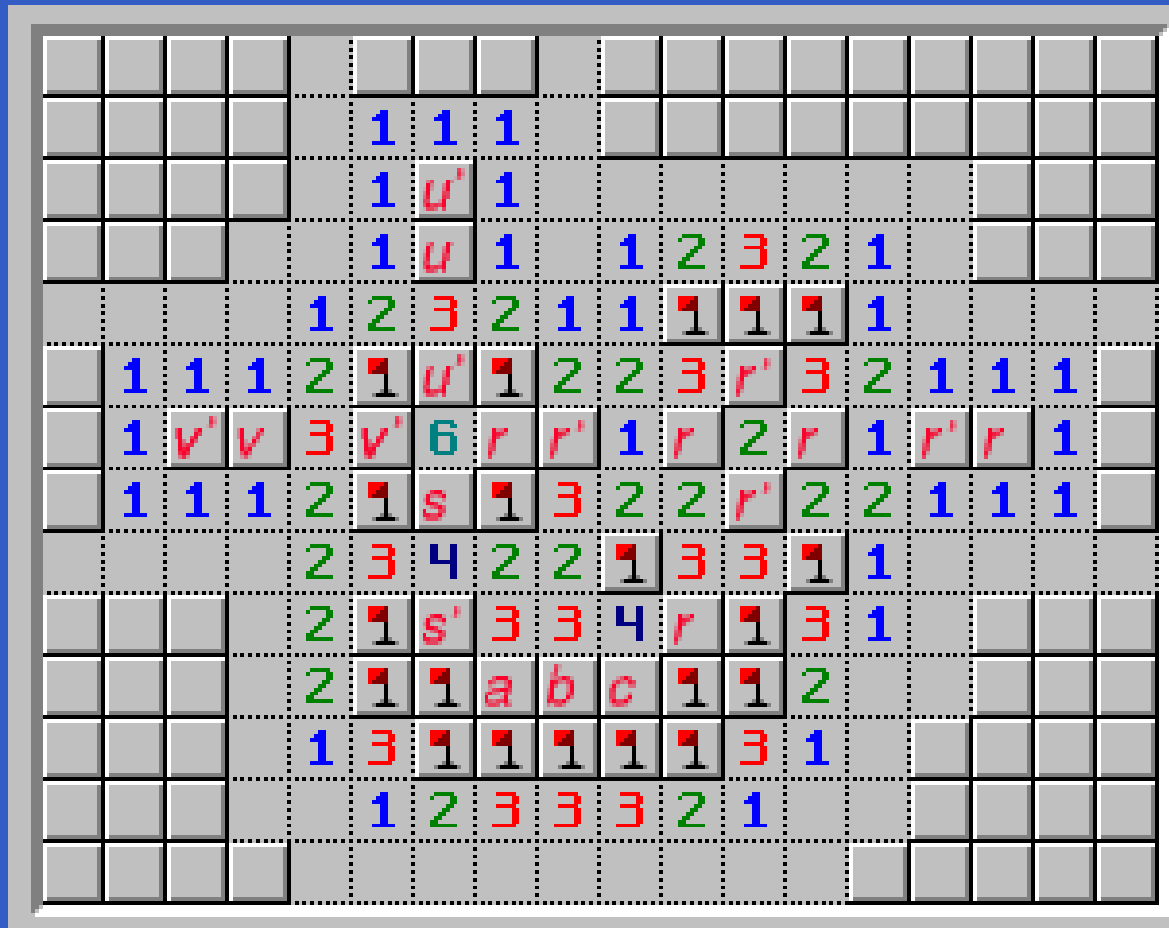
…with the internal wires shown.

An OR gate

If *r* is free...

An OR gate

If *r* has a mine (case 1)...

An OR gate

If *r* has a mine (case 2)…

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- Logic gates: NOT AND OR XOR, etc.

NP-completeness

- We can build any logic circuits in minesweeper, so. . .

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- **The Minesweeper Consistency Problem is NP-complete.**

Some interesting NP problems

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- Determining a computer user's secret key from his public key

The P equals NP question

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- It would suffice to show
  - *either* that the minesweeper consistency problem *has* and efficient solution
  - *or* that there is no such algorithm for the minesweeper consistency problem